

Please go through each slide stopping until you have understood the concept described

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# Calculus was invented simultaneously by Newton in England and Leibnitz in Germany.

Newton is better known for his theories of gravity



Sir "Irate" Newton

Well, you would not be happy if you had just been hit on the head by an apple



# 29 - Sum Calculus

Calculus has two operators called "Differentiation" and "Integration" related to a curve or graph of a formula such as  $y=x^2$ .

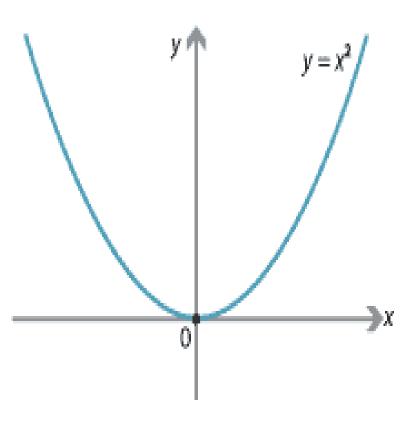
Differentiation is used to find the slope of the tangent to the curve which is also the rate of change of the curve.

For instance, if a rocket's speed is given by  $y=x^2$ , where y is the speed and x is the time. The acceleration of the rocket is given by y=2x.

Integration is the opposite, i.e. if the slope pf the curve is given by the formula y=2x the curve is  $y=x^2$ .

So how do we prove this.

We use a small changes in x and y called "delta x" and "delta y" when we use the Greek symbol for delta =  $\delta$ .





#### At the points x and $x+\delta x$ on the graph.

First draw the points x,y and  $x+\delta x$ ,  $y+\delta y$  and then draw the right angled triangle in red.

The slope of the line connecting the two points is given by the formula  $\delta y$  /  $\delta x$ 

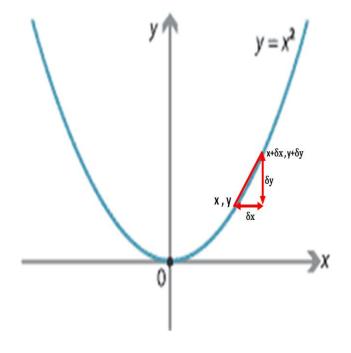
But 
$$\delta y = (x + \delta x)^2 - x^2 = x^2 + 2x \delta x + \delta x^2 - x^2 = 2x \delta x + \delta x^2$$
  
Now divide both sides of the equation by  $\delta x$ .

$$\delta y/\delta x = 2x + \delta x$$

If the two points were actually almost same point then  $\delta x$  is tending towards zero so we can eliminate it. When  $\delta x$  is zero we rewrite  $\delta y/\delta x$  as dy/dx.

Then we have dy/dx = 2x

So the slope of the curve at point x.y is dy/dx = 2x



#### Sums. The're Math Magic

The rule for differentiation is to take the index of x as the multiplied of x with the new index as one less than before.

e.g. if  $y=x^4$  then  $dy/dx = 4x^3$  and:

e.g. if 
$$y=x^4 + x^3 + x^2 + x$$
 then  $dy/dx = 4x^3 + 3x^2 + 2x + 1$ 

(the 1 is there because  $x = x^1$ )



## For integration we use the symbol \

Now Integration is the opposite and to so this we generally remember the way we did the differentiation.

e.g.  $\int$  of  $4x^3 + 3x^2 + 2x + 1$  is  $x^4 + x^3 + x^2 + x + c$  where c is some constant or number.

Why? Because if  $y = x^4 + x^3 + x^2 + x + c$  then  $dy/dx = 4x^3 + 3x^2 + 2x + 1 + 0$  because  $c = cx^0$  which means we have to multiply c by zero and because  $x^0 = 1$ 

There is one extra rule about constants (numbers) which are kept.

e.g. if  $y=ax^4$  then  $dy/dx=4ax^3$ 

and if  $y=ax^4 + 7$  then  $dy/dx = 4ax^3 + 7$ 



There is a web-site for differentiation at

https://www.derivative-calculator.net/

However you need to write  $y=ax^4$  as  $y=ax^4$  where ^ is used to indicate the index of x.

For integration please see

https://www.integral-calculator.com/

#### Sums. The're Math Magic

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